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ESTIMATION OF STRONG-MOTION ACCELERATION APPLYING POINT SOURCE MODELS

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ABSTRACT

A theoretical ground motion model based on seismic source models has been applied to the available strong-motion acceleration recordings in Iceland. The source parameters have been estimated from the acceleration records and different estimation methods are studied. The applied model is used to study the characteristics of strong ground motion with the main objective of improving models for seismic hazard studies. The applied ground motion model is useful for describing the attenuation of ground motion parameters such as peak ground acceleration, root mean squared acceleration and spectral acceleration. The model can also be used for simulating realistic input records for computational structural models using a stochastic approach. A comparison of the results for the attenuation of the ground motion parameters obtained from the Icelandic data is compared with results from North-America and Europe by applying the theoretical model to other datasets as well as considering attenuation relations found in the literature. The models are applied to specific earthquakes as well as earthquakes with similar magnitude and similar source mechanism. Special consideration is given to near field acceleration as well as rate of attenuation.

Introduction

Over the past decades many empirical attenuation relations have been presented with the purpose of scaling strong-motion acceleration for engineering purposes. These attenuation relations are in most cases similar in form, with magnitude and distance from source to site as the independent variables. The parameters are estimated by fitting the relations to the data (in most cases PGA) by means of regression analysis. The disagreement is apparent between the various attenuation relations that have been put forward in the literature (see, for example (Douglas 2003) for a recent review). This disagreement is partly due to the difference between the data sets from which the models are derived and also due to different modeling, processing and estimation techniques.

The attenuation model used in this paper is derived theoretically, although it has model parameters that can be determined empirically. It is based on Brune's near- and far-field models; see (Brune 1971) and is extended with an exponential term to account for spectral decay at high frequencies (Ólafsson and Sigbjörnsson 1999). The theoretical model is derived using Parseval's

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theorem and has also been derived for response spectra (Snæbjörnsson, Sigbjörnsson, and Ólafsson 2004). The modeling approach used here has been called the stochastic method (Boore 2003) and is based on the work of Hanks and McGuire (Hanks and McGuire 1981) and was later generalized by Boore (Boore 1983).

In this paper the theoretical model, for both near- and far-field, is presented and then studied using data from shallow strike-slip earthquakes in Iceland, June 2000, of magnitude M_S 6.6 and 6.5 (see (Thórarinsson et al. 2002) and (Sigbjörnsson, Ólafsson, and Thórarinsson 2004)). The model is also applied to PGA from 124 accelerograms obtained in several earthquakes in Europe and North-America with magnitudes in the range 6.3 to 6.7. The results are finally compared and discussed.

Strong-motion model

For the estimation of strong motion, theoretical models derived by Brune (Brune 1970) have been applied. The original Brune models have been augmented with an exponential function that accounts for the spectral attenuation at high frequencies. The strong motion estimation can be represented by two models one for the near-field and one for the intermediate- and far-field. The magnitude spectra for the near-field can be written as follows (Sigbjörnsson and Ólafsson 2004):

$$\left|A(\omega)\right| = \frac{7}{8}\frac{C_p M_o}{\rho\ \ \beta\ \ r^3}\frac{\omega}{\sqrt{\omega^2+\tau^{-2}}}\exp(-\tfrac{1}{2}\kappa_o\omega) \tag{1}$$

where C_p is the partitioning factor, ρ is the material density of the crust, M_o is the seismic moment, r is the radius of the circular fault, τ is the rise time, β is the shear wave velocity and κ_o is the near-field spectral decay parameter.

For the far-field the amplitude spectrum can be represented by the following equation:

$$\left|A(\omega)\right| = \frac{2C_P R_{\theta\phi} M_o}{4\pi\beta^3\rho \mathcal{R}}\frac{\omega^2}{(1+(\omega/\omega_c)^2)}\exp(-\tfrac{1}{2}\kappa\omega) \tag{2}$$

Here, $\mathcal{R}$ is geometric spreading function representing the distance from source to site, ω_c is the corner frequency, $R_{\theta\phi}$ denotes the radiation pattern. The distance from source to site, $\mathcal{R}$, is modeled by the following expression

$$\mathcal{R} = \begin{cases} D_2^{1-n}\ \ D^n & D_1 < D \le D_2 \\ \quad D & D_2 < D \le D_3 \end{cases} \tag{3}$$

here $D = \sqrt{d^2+h^2}$, d is the epicentral distance and h is a depth parameter. In this case h should not be interpreted as a hypocentral depth but ought to be taken as a scaling factor. The parameters D_1, D_2 and D_3 are used to set the limits for the different zones of the spreading function. The parameter n is in the range 1 to 2 but is assumed to have the value 1.41 in the present study.

Time series representing strong ground acceleration can be simulated applying these models by using the stochastic method. The models have been calibrated by the available strong-motion acceleration measured in Icelandic earthquakes. A strong-motion measurement network has been operated since 1986 in Iceland and 3300 accelerograms have been obtained in 300 earthquakes. The largest earthquakes are the 1987 Vatnfjöll earthquake (M_w = 6.0) and the South-Iceland earthquake in the year 2000 (M_w = 6.5) (Sigbjörnsson, Ólafsson, and Thórarinsson 2004). Strong-motion acceleration data from several of the largest earthquakes are available in the Internet Site for European Strong-Motion Data (Ambraseys et al. 2002).

Attenuation of ground motion

A theoretical relation for the attenuation of ground motion can be dervived using the far-field Brune model (Ólafsson 1999) using Parseval theorem. The theoretical attenuation relation can be written as follows, where a_{rms} are the rms-value of the ground acceleration:

$$\log_{10}(a_{\mathrm{rms}}) = \log_{10}\left(\frac{1}{\sqrt{\pi}} \left(\frac{7}{16} \right)^{1/3} \frac{2\mathrm{C_P} \left\langle \mathrm{R}_{\theta\phi} \right\rangle \Delta\sigma^{2/3}}{\beta\rho\sqrt{\kappa}} \right) + \frac{1}{2}\log_{10}\left(\frac{\Psi}{T_\mathrm{d}} \right) + \frac{1}{3}\log_{10}(M_\mathrm{o}) - \log_{10}(\mathrm{R}) \tag{4}$$

here T_d represent the strong motion duration, $\Delta\sigma$ is the seismic stress drop and Ψ represents a dispersion function of the variable $\lambda = \kappa\omega_c$, and can be evaluated by a closed form expression (Sigbjörnsson and Ólafsson 2004). The peak ground acceleration can be evaluated as $a_{\mathrm{peak}} = pa_{\mathrm{rms}}$ by using a peak factor p obtained by applying the theory of locally stationary Gaussian processes (Vanmarcke and Lai 1980).

The time domain properties of acceleration, a, can readily be derived from the Fourier spectrum, A, by applying the Parseval theorem. This gives:

$$I = \int_0^T a^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|A(\omega)\right|^2 d\omega \tag{5}$$

The last integral can be evaluated after substituting Eq. 2. The result is:

$$I = \frac{1}{\pi}\left(\frac{7}{16}\right)^{2/3} \left(\frac{2C_P \left\langle R_{\theta\phi} \right\rangle \Delta\sigma^{2/3}}{\beta\rho R} \right)^2 \frac{\Psi}{\kappa} M_\mathrm{o}^{2/3} \tag{6}$$

where Ψ denotes a dispersion function given by the following integral for which a closed form solution is readily obtained:

$$\Psi = \lambda \int_0^{\infty} \frac{\varpi^4}{(1+\varpi^2)^2} \quad e^{-\lambda\varpi} d\varpi \tag{7a}$$

$$\Psi = 1 - \frac{1}{2}\lambda ci(\lambda)\left(\lambda\cos(\lambda) + 3\sin(\lambda)\right) - \frac{1}{2}\lambda si(\lambda)\left(\lambda\sin(\lambda) - 3\cos(\lambda)\right) \tag{7b}$$

Here, ci(·) and si(·) represent the cosine and sine integrals, respectively, $\varpi = \omega/\omega_c$ and:

$$\lambda = \kappa\omega_c \tag{8}$$

where ω_c is the corner frequency. The sine and cosine integrals applied in Eq. 7 are given, respectively, as follows:

$$\begin{aligned} si(\lambda) &= -\frac{\pi}{2} + \int_0^{\lambda} \frac{\sin(t)}{t} dt \\ ci(\lambda) &= \gamma + \ln(\lambda) + \int_0^{\lambda} \frac{\cos(t)}{t} dt \end{aligned} \tag{9}$$

where γ is the Euler constant ($\gamma \approx 0.5772$).

Near-field approximation

The model described in the previous section is not valid in the near-field and can, therefore, not be expected to describe the peak ground acceleration accurately close to the fault. To be able to obtain an approximation valid for shear waves in the near-fault area, it is suggested that the Brune near-field model, Eq. 1, is used. An approximation for the RMS and PGA is now obtained by applying the Parseval theorem and, then, carrying out the integration. The result is:

$$\log_{10}(a_{rms}) = \log_{10}\left(\frac{1}{\sqrt{\pi}} \frac{7}{8} \frac{C_p}{\rho\beta r^3 \sqrt{\kappa_o}}\right) + \frac{1}{2}\log_{10}\left(\frac{\Psi_o}{T_o}\right) + \log_{10}(M_o) \tag{11}$$

Here, the duration is denoted by T_o and Ψ_o is a dispersion function given as:

$$\Psi_o = \lambda \int_0^{\infty} \frac{\varpi^2}{1+\varpi^2} e^{-\lambda\varpi} d\varpi \tag{12a}$$

$$\Psi_o = 1 - \lambda\left(ci(\lambda)\sin(\lambda) - si(\lambda)\cos(\lambda)\right) \tag{12b}$$

where $\lambda = \kappa_o / \tau$. It is seen that the PGA predicted by this equation is independent of the epicentral distance and hence should give an estimate on the upper-bound of PGA. Another result, which emerges when the equation:

$$\Delta\sigma = \frac{7}{16}\ \frac{M_o}{r^3} \tag{13}$$

is substituted into Eq. 11, is that the RMS acceleration is directly proportional to the stress drop. That is:

$$a_{rms} = \frac{2}{\sqrt{\pi}}\frac{C_p\,\Delta\sigma}{\rho\beta\sqrt{\kappa_o}}\sqrt{\frac{\Psi_o}{T_o}} \tag{14}$$

This indicates that assuming constant stress drop the ground acceleration in terms of the rms value can decrease with increasing earthquake magnitude.

The Brune near-field model is an approximation that can only be applied to moderate sized shallow strike-slip earthquakes, preferably when the fracture is extended up to the free surface. Under these assumptions the presented simplified model yields results in fair agreement with the near-fault formulas presented by Ambraseys and Douglas (Ambraseys and Douglas 2003).

Earthquake response spectrum

The rms response as well as peak response of linear elastic system can be obtained within the framework of the theory described here above.

Far-field approximation

Application of the above described models lead to the following expression after the integration has been carried out:

$$x_{rms}(t) \approx \frac{1}{\omega_o^2}\sqrt{I_F + \frac{1}{\pi T_d}\left|A_F(\omega_o)\right|^2\left(\frac{\pi\omega_o}{4\zeta} - 1\right)} \tag{15a}$$

where ω_o denotes the undamped natural period, ζ is the critical damping ratio, T_d is the duration $|A_F|$ is given by Eq. 2 and

$$I_F = \frac{1}{\pi}\left(\frac{7}{16}\right)^{2/3}\left(\frac{C_P\left\langle R_{\theta\phi}\right\rangle \Delta\sigma^{2/3}}{\beta\rho R}\right)^2 \frac{\Psi}{T_d\,\kappa} M_o^{2/3} \tag{15b}$$

Here the same notation is applied as before, $\Delta\sigma$ is stress drop and Ψ denotes a dispersion function given in Eq. 7.

Near-field approximation

The model described in the previous section is not valid in the near-field and can, therefore, not be expected to describe the response accurately close to the fault. To obtain an approximation which is valid for shear waves in the near-fault area the Brune near-field model can be used. Hence, the near-field acceleration spectrum is approximated as given in Eq. 1, after modifying the high frequency part with an exponential term and accounting for the free surface and partitioning of the energy into two horizontal components. Then the following expression is obtained after the integration has been carried out:

$$x_{rms}(t) \approx \frac{1}{\omega_o^2}\sqrt{I_N + \frac{1}{\pi T_o}\left|A_N(\omega_o)\right|^2\left(\frac{\pi\omega_o}{4\zeta} - 1\right)} \tag{16a}$$

where ω_o denotes the undamped natural period, ζ is the critical damping ratio, T_o is the source duration $|A_N|$ is given by Eq. 1 and

$$I_N = \frac{1}{\pi}\left(\frac{7}{8}\frac{C_p}{\rho\beta r^3}\right)^2 \frac{\Psi_o}{T_o\,\kappa_o} M_o^2 \tag{16b}$$

Here, the source duration is denoted by T_o and Ψ_o is a dispersion function given in Eq. 12.

Peak response

The peak response can be obtained applying the random vibration theory as outlined by Vanmarcke and Lai (Vanmarcke and Lai 1980). Hence, introducing the peak factor, p, the response spectrum for a linear elastic sdof system can be expressed as follows:

$$S_D(\omega_o, \zeta) = \max_{t\in T} x(t) = p \cdot x_{rms}(t) \tag{17}$$

It should be noted that the peak factor will generally be a function of the duration and effective frequency and bandwidth of the system, in other words, it depends on the effective number of peaks within the time window considered. Furthermore, the peak factor depends on the probability of exceedance referred to the time window under consideration. However, in the following a median value is used for the peak factor, which is close to the most probable value, corresponding to positive zero crossings:

$$p \cong \sqrt{2\ln\left(2.8 T_{\mathrm{d}} f_o / 2\pi\right)} \tag{18}$$

where f_o is the natural frequency of the system. A thorough treatment of the peak factor is given by Vanmarcke and Lai (Vanmarcke and Lai 1980).

Numerical results

In the following the presented model is compared with two different datasets: (1) Icelandic strong-motion data from South Iceland earthquakes in June 2000 and (2) shallow earthquakes from Europe and North-America that are chosen according to the conditions of depth < 15 km and any magnitude in the range 6.3 to 6.7. The records chosen are from rock or stiff soil sites.

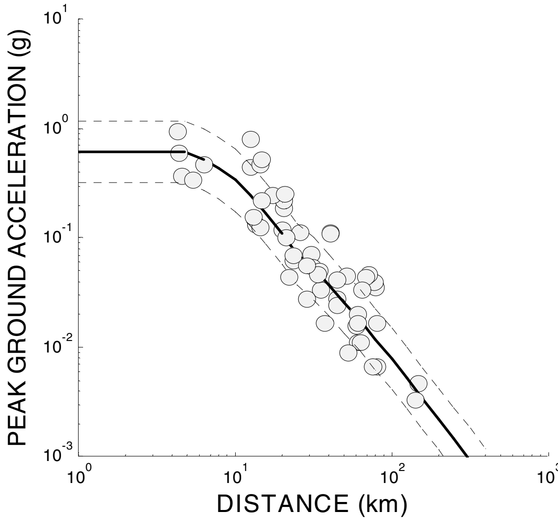
The data from the Icelandic earthquakes is composed of 98 horizontal components of accelerations from two earthquakes occuring on June 17th (M_S 6.6) and June 21st (M_S 6.5) 2000. The data from the earthquakes are available in the ISESD database (Ambraseys et al. 2002). The data from the European earthquakes are obtained from ISESD and the North American data is obtained from the Pacific Earthquake Engineering Research Centre strong-motion database (PEER 2005). A total of 124 records of horizontal acceleration come from these two regions. The earthquakes are listed in Table 1. The focal mechanism for the Icelandic earthquakes is strike-slip but for the European and North-American earthquakes chosen here the mechanism is normal and oblique in addition to strike-slip.

Table 1. The earthquakes used in the study that are originated in Europe and North-America.

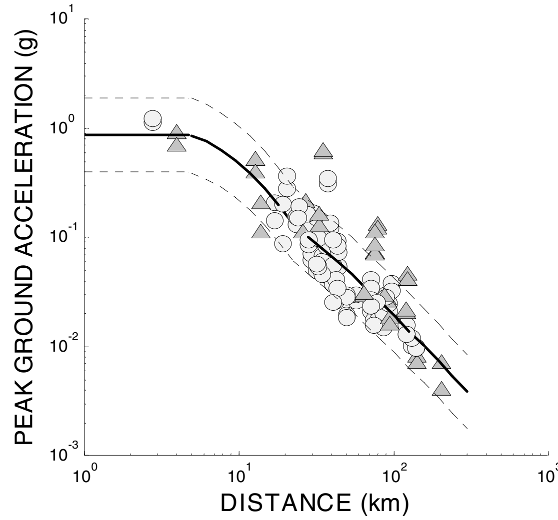
Event	**Country**	**Date / Time**	**Magnitude**	**Number of records**
Borrego Mtn	USA	9 April 1968 / 02:30	6.5	4
San Fernando	USA	9 February 1971 / 14:00	6.6	14
Friuly	Italy	6 May 1976 / 20:00	6.5	6
Imperial Valley	USA	15 October 1979 / 23:16	6.5	6
Victoria	Mexico	9 June 1980 / 03:28	6.4	2
Coalinga	USA	2 May 1983 / 23:42	6.5	36
Magion Oros	Greece	6 August 1983 / 15:43	6.6	6
Panisler	Turkey	30 October 1983 / 04:12	6.6	2
Superstition Hill	USA	24 November 1987 / 13:16	6.7	2
Erzincan	Turkey	13 March 1992 / 17:18	6.6	6
Kozani	Turkey	13 May 1995 / 08:47	6.5	16
Aigion	Greece	15 June 1995 / 00:15	6.5	16
Adana	Turkey	30 October 1998 / 13:55	6.3	8

Figs. 1a) and 1b) show the PGA values from the two datasets plotted with respect to distance from source. Fig. 1a) shows the June 2000 earthquakes in Iceland, with the solid curve

representing the mean values calculated from the theoretical model and the dotted curves the mean values + / – one standard deviation. Fig. 1b) shows the PGA values of the North-American and European data plotted with respect to distance. The triangles represent earthquakes with a strike-slip source mechanism and the filled circles represent earthquakes with other types of source mechanisms.



a)



b)

Figure 1. a) PGA attenuation of the June 2000 earthquakes in Iceland. b) PGA attenuation of several earthquakes from Europe and North-America with magnitudes in the range 6.3 to 6.7. The triangles represent strike-slip earthquakes and the filled circles represent earthquakes with other types of source mechanisms. In both figures the solid curve represents the mean values calculated from the theoretical model and the dotted curves represent the mean values + / – one standard deviation.

Discussion and conclusions

The theoretical model is seen to fit the PGA data from the European and North-American region equally well as the data from the Icelandic earthquakes, as can see by comparing Figs. 1a) and 1b). The same form of the attenuation curve is seen to fit both data sets equally well. The curve for the Icelandic data is however lower by a factor $\sqrt{2}$ compared with the curve for the North-American and European data. This discrepancy can partly be attributed to difference of the depth in the crust where the earthquakes originate. On the average the Icelandic events occur on shallower depths than the North-American and European events. Regional differences of crustal properties can also be a source of discrepency as reflected in parameters such as stress drop, $\Delta\sigma$, and spectral decay, κ. There is however a trade-off between the model parameters $\Delta\sigma$ and κ (Boore, Joyner, and Wennerberg 1992). Different pair of $\Delta\sigma$ and κ can therefore give the same results. Instead of lowering the κ values the radius could be decreased, which increased the stress drop.

The source parameters have been estimated for the two Icelandic earthquakes using the acceleration data with an estimation procedure similar to the procedure described in (Ólafsson 1998) and (Ólafsson 1999). They agree well with the parameters used here as model parameters for the attenuation model. The parameters from the other data have not been computed by the same procedure. We know, however, that values of $\kappa = 0.03$ s to 0.04 s and $\Delta\sigma = 70$ -100 bar are found to give a good fit to the North-American data (Atkinson and Boore 1995). The standard deviation, σ, was smaller for the Icelandic earthquake $\sigma = 0.2830$ compared with $\sigma = 0.3412$ for the North-American and European data-set. The non-Icelandic data is also seen to have a few outliers. The outliers are seen to bee around epicentral distances of 40 and 120 km. The outliers can possibly be due to "Moho bounce" (Sommerville and Yoshimura 1990; Douglas 2001).

Several of the empirical attenuation relations that have been presented in the literature have been applied to the Icelandic data from the June 2000 earthquakes, and have been found to give a poor fit to the data (Ólafsson 1999). This has also been the observation for Icelandic earthquakes with lesser magnitudes (Sigbjörnsson 1990). The empirical attenuation relations are also found to be dependent on geographical regions, where the data originates, which they are based on. In addition to this the empirical equations do often not fit well to specific earthquakes. This is not surprising considering the fact that the regression parameters are often estimated from earthquakes over a wide magnitude range.

More research is needed to refine the theoretical modelling of attenuation, as is presented here, in order to improve its usefulness for hazard assessment. Application to additional data is important and grouping the data according to type of earthquake mechanism, soil conditions etc., in order to estimate model parameters and determine attenuation of ground motion.

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